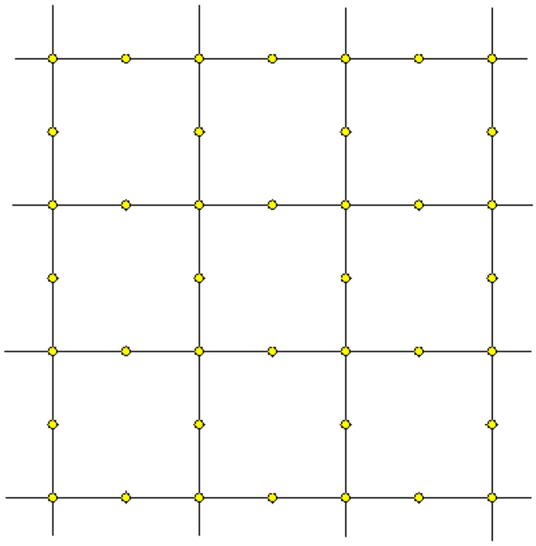
Bravais Lattices

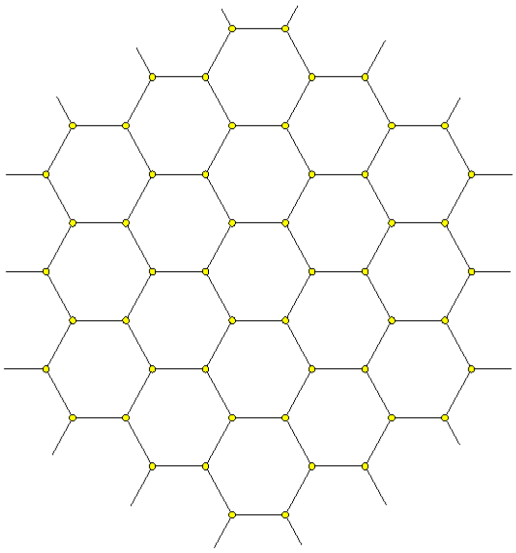
Definitions

A Bravais lattice is an infinite array of points such that the lattice looks *exactly* the same from every lattice point.

The following is *not* a Bravais lattices because the lattice points do not have identical neighbours.



Nor is this because, although every point has identical neighbours, at alternate lattice points, the lattice is inverted.



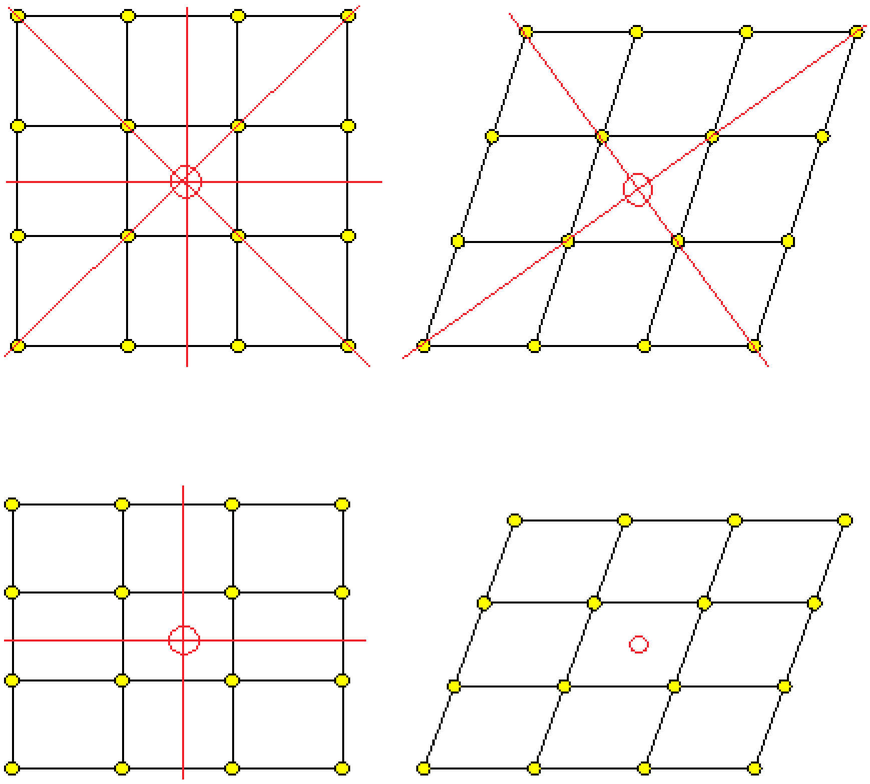
In each Bravais lattice of 2 or more dimensions, there are an infinite number of different directions in which the lattice may be moved so that it coincides with itself after being moved a given distance. There will be a small number of directions in which the repeat distance is minimum. These directions are called the *principal axes* and we can choose 2 (or more) to define the lattice

Bravais Lattices in 2 dimensions

We can start by listing 4 categories:

1. Square: $a = b$, $\theta = 90°$
2. Rhomboid: $a = b$, $\theta \neq 90°$
3. Rectangular: $a \neq b$, $\theta = 90°$
4. Oblique: $a \neq b$, $\theta \neq 90°$

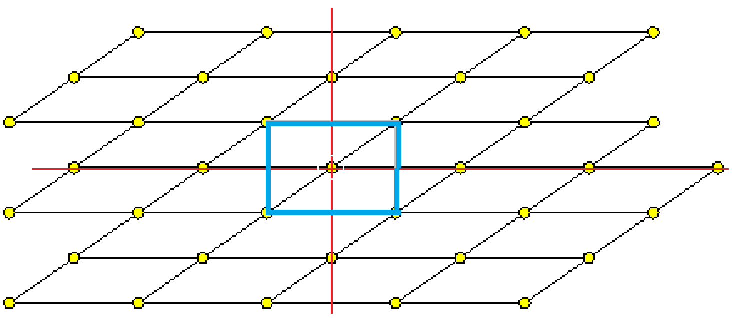
which look like this:

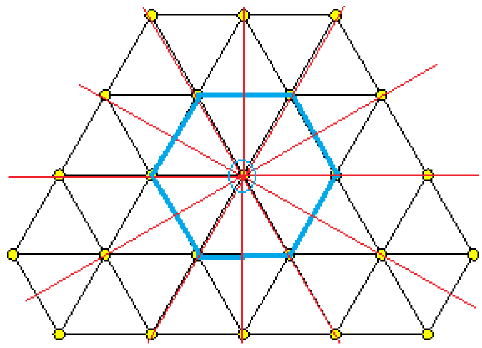


It can be seen that the four lattices are characterised by different degrees of symmetry. The square lattice has four reflection planes and one 4-fold axis of rotation; the rhomboid lattice has two reflection planes and one 2-fold axis of rotation; the rectangular lattice also has two reflection planes and one 2-fold axis of rotation but they are not in the same direction as those in the rhomboid lattice; the oblique lattice has the rhomboid lattice has no reflection planes and only one 2-fold axis of rotation.

There are, however, two further special cases to consider.

5. Oblique/rectangular: $a \neq b$, $\theta = \arccos(a / 2b)$
6. Hexagonal: $a = b$, $\theta = 120°$





Owing to the unique relation between a, b and θ in the first case, the top lattice points in the next layer above makes a rectangle with the bottom points in the first layer and therefore has the same symmetry as the rectangular lattice. The difference is that the rectangular cell has an additional lattice point in the centre of the rectangle.

An inspection of the rhomboid lattice will reveal that it, too, has a centred rectangle buried within it so the oblique/rectangular and the rhomboid lattices are actually exactly the same, differing only in the choice of axes we make.

The hexagonal lattice is, however, completely new because it has six reflection planes and one 6-fold axis of rotation. This puts it into a completely different class.

It only remains to show that there no other lattices are possible. Are there, perhaps, lattices with more than two lattice points per unit cell? Can we put lattice points in the centres of the other cells?

I do not have a formal proof but the answer to these two questions is – no.

If we put another lattice point in the centre of a square, we simply get a smaller square lattice and a lattice point at the centre of a rhombus is not identical to the points at the corners.

The final list

Bravais lattices in 2 dimensions therefore fall into four symmetry groups, one of which has two forms. They are, in decreasing order of symmetry:

- Hexagonal
- Square
- Rectangular;   Centred rectangular
- Oblique

Bravais Lattices in 3 dimensions

In 3 dimensions, the three principal axes a, b and c can abe equal or different in length and the angles between them α , β and γ can be equal, or different, or some fraction of a circle. All this amounts to a huge number of different possibilities but, as we have seen in the 2 dimensional cases, what is more important is the symmetry class to which these different cases belong.

It turns out that there are 7 basic types of symmetry as follows:

1. Cubic $a = b = c,\ \alpha = \beta = \gamma = 90°$
2. Tetragonal $a = b \neq c,\ \alpha = \beta = \gamma = 90°$
3. Orthorhombic $a \neq b \neq c,\ \alpha = \beta = \gamma = 90°$
4. Monoclinic $a \neq b \neq c,\ \alpha = \beta,\ \gamma \neq 90°$
5. Trigonal $a = b = c,\ \alpha \neq 90°,\ \beta \neq 90°,\ \gamma \neq 90°$
6. Triclinic $a \neq b \neq c,\ \alpha \neq 90°,\ \beta \neq 90°,\ \gamma \neq 90°$
7. Hexagonal $a = b = c,\ \alpha = 60°,\ \beta = \gamma = 90°$

(The trigonal system is also called rhombohedral)

In addition, we must consider the addition of further lattice points within the primitive cell which do not break the symmetry of the system.

There are several ways in which extra lattice points can be added to a primitive cell. These are (in addition to the primitive cell P):

I	Body-centred	An extra lattice point in the centre of the cell
F	Face-centred	6 extra lattice points placed in the centres of each face
C	End-centred	2 extra lattice points placed in the centres of the end faces

Eliminating those which do not produce new distinct lattices, we are left with just 14 3 dimensional Bravais lattices.