The Physics of Flight (3) - Hovering

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Abstract

In 1934 the French etymologist August Magnan wrote in the introduction to his book '***Le Vol Des Insects***' that it was aerodynamically impossible for a honey bee to fly. In 1984 C.P.Ellington[1] published a seminal series of articles which seemed to lend support to the idea that insects were performing aerodynamic feats which could not be easily explained, and this resulted in an explosion of interest in insect and bird flight which continues unabated to this day. Recent advances in experimental and computational techniques have enabled us to measure, visualize and calculate the flow round an insect's wings in ever greater detail and for a comprehensive summary of the present state of research into the area I would recommend a review paper by Sane[2]. He describes at least four effects which purport to increase the amount of lift that would be expected on the basis of conventional aerodynamics. But how bad was the old 'back of the envelope' calculation? Is it really necessary to invoke such complicated mechanisms to explain something which happens before our very eyes every day of the week? If the old calculations give an answer within an order of magnitude, I would be happy with that. If the old calculations are more than a factor of 10 out, no amount of tweaking with 'delayed stall' or 'wake capture' will make up the deficit and we will have to conclude that the flight of insects and hummingbirds is literally a miracle. Hopefully it will not come to that. This article attempts to find out.

Hovering insects and birds

Insects can hover. So can hummingbirds. Even small birds such as sparrows and finches can hover briefly and pigeons can take off vertically, albeit with great difficulty. Slow-motion photography of a hummingbird hovering clearly shows the wings in an almost vertical position, flapping backwards and forwards at incredibly high speed. As the wing moves forwards, the wing is inclined facing downwards (pronation) but when the wing moves backwards, the wing is rotated so that it is upside down (supination) generating lift on the backstroke as well as on the forward stroke. A pigeon does exactly the same thing. You can actually see the wings moving vigorously forwards and backwards and you can sometimes hear the clap of the wings as they hit each other at the extremities of the stroke. Many insects also use the same technique.

But how much lift (defined here as the vertical force) can a flapping wing generate in still air? The conventional analysis which we have employed in the previous two articles cannot be used here because the angle of attack of the wing is typically greater then 45° and the flow past the wing is far from laminar. Nevertheless, it is possible, using some simple assumptions, to derive an expression for the lift generated which produces results which are at least consistent with observed behaviour.

Fig. 1

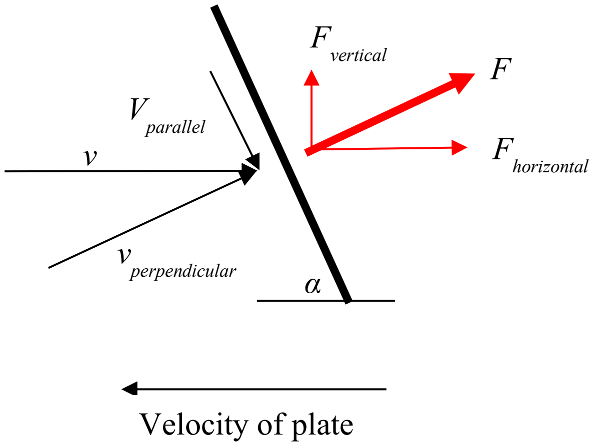


Figure 1 shows a flat plate of area S_W inclined at an angle α to the horizontal moving horizontally through air of density ρ at a speed v. The velocity of the air impinging on the plate may be resolved into two components, one at right angles to the plate $v_{perpendicular}$ and the other parallel to it $v_{parallel}$. The perpendicular component causes a force F to act on the plate but if we neglect viscous forces, the parallel component does not exert a significant force on the plate.

The force exerted on a flat plate by a stream of fluid is easy to calculate from the momentum destroyed every second. It is simply

$$F = S_W \rho \, v_{perpendicular}{}^2 \tag{1}$$

Since

$$v_{perpendicular} = v \sin(\alpha) \tag{2}$$

it follows that

$$F = S_W \rho \, v^2 \sin^2(\alpha) \tag{3}$$

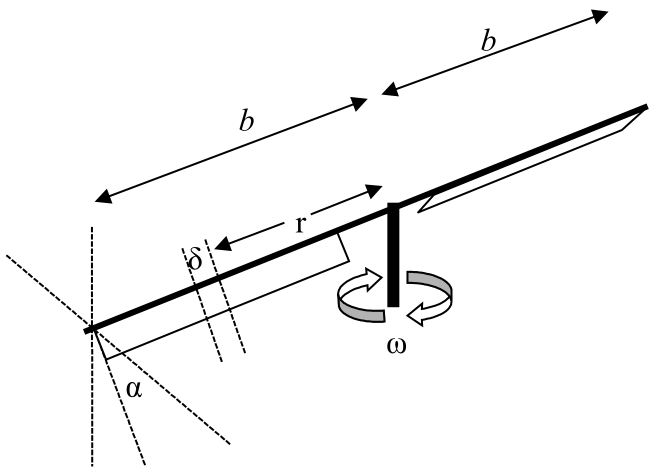
We can now write down the vertical and horizontal components of F (which we shall now refer to as *Lift* and *Drag*) thus:

$$F_{lift} = S_W \rho \, v^2 \sin^2(\alpha) \cos(\alpha) \tag{4}$$

$$F_{drag} = S_W \rho \, v^2 \sin^3(\alpha) \tag{5}$$

To test this theory we might imagine constructing a bizarre form of helicopter with two almost vertical blades rotating round and round at an angular speed ω as illustrated below.

Fig. 2



To calculate the lift produced we must integrate along the length of the blade. Consider a small element of the blade at a distance r from the axis and of length δr Its horizontal velocity through the air will be $v = r\omega$ where ω is the angular velocity of the blade. Using exactly the same arguments as before (ie equations (4) and (5)) the components of lift and drag on this element are

$$\delta F_{\text{lift}} = c\,\rho\, r^2\omega^2 \sin^2(\alpha)\cos(\alpha)\,\delta r \tag{6}$$

$$\delta F_{\text{drag}} = c\,\rho\, r^2\omega^2 \sin^3(\alpha)\,\delta r \tag{7}$$

where c is the width (chord) of the blade.

To find the total lift on the blade we must integrate from 0 to b along each half of the blade.

$$F_{\text{lift}} = 2\int_0^b c\rho\, r^2\omega^2 \sin^2(\alpha)\cos(\alpha)\,dr \tag{8}$$

from which we obtain (putting $\omega = 2\pi f$)

$$F_{\text{lift}} = (8\pi^2/3)\, b^3\, c\,\rho\, f^2 \sin^2(\alpha)\cos(\alpha) \tag{9}$$

Now since, for a hovering machine, F_{lift} must be equal to the weight of the machine Mg

$$f = \sqrt{\frac{3Mg}{8\pi^2 b^3 c\rho \sin^2(\alpha)\cos(\alpha)}} \tag{10}$$

To fix our ideas more firmly, imagine a model paddlewheel ‘copter of total mass $M = 1.5$ kg with a rotor blades of length $b = 0.3$ m and blade width $c = 0.04$ m, angled at $\alpha = 60°$. The density of air is 1.3 kg m^{-3}. In order to get off the ground, equation (10) tells us that the rotor frequency would have to be of the order of 33 Hz or 2000 rpm.

This doesn’t sound impossible, so why aren’t all helicopters designed this way? The answer lies in the excessive amount of power needed to drive such a system. To calculate this we must integrate the power lost by each element along the blade. This is equal to $F_{\text{drag}}.r\omega$

$$\text{Power}_{\text{loss}} = 2\int_0^b c\rho\, r^3\omega^3 \sin^3(\alpha)\,dr \tag{11}$$

from which we obtain

$$\text{Power}_{\text{loss}} = 4\pi^3 b^4 c\,\rho\, f^3 \sin^3(\alpha) \tag{12}$$

Putting in our proposed figures gives a power requirement of 1200 W. This is, I imagine, rather greater than the feasible power output of any known motor weighing less than 1.5 kg.

(It is actually more convenient to deduce an expression for the Lift/Power ratio:

$$R_{LP} = \frac{2\cot(a)}{3\pi\; b\; f} \tag{13}$$

and then use

$$\text{Power}_{\text{loss}} = \frac{Mg}{R_{LP}} \tag{14}$$

which gives us $R_{\text{LP}} = 0.012$ N W^{-1} and $\text{Power}_{\text{loss}} = 1200$ W as before. This method also has the advantage of emphasizing the very poor lift to power ratio.)

Conventional helicopters

To put this in perspective we can calculate the power requirements of a conventional helicopter design. In the first article in this series, we showed that the rotor frequency was given by the expression:

$$f = \sqrt{\frac{3Mg}{4\pi^2 C_L b^3 c\rho}} \tag{15}$$

If we keep the dimensions of the blades the same but give them a suitably aerodynamic shape and an appropriate angle of attack such that the coefficient of lift C_L is equal to 0.5, we can turn our paddlewheel 'copter into a proper helicopter where $b = 0.3$ m and $c = 0.04$ m. This calculates to 40 Hz or about 2400 rpm.

We also showed that the power required to drive the rotors was given by the expression:

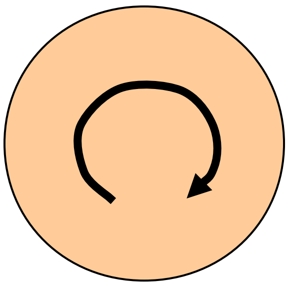
$$\text{Power loss} = \sqrt{\frac{M^3 g^3}{2\pi\, b^2 \rho}} \tag{16}$$

which evaluates to a mere 68 W. A much more reasonable figure!

Insects and birds

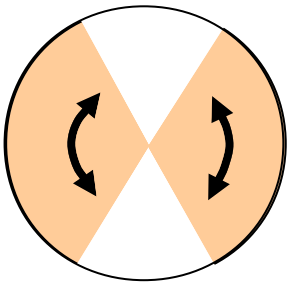
You may be surprised to hear that insects and birds do not rotate their wings like a helicopter! But our analysis of the paddlewheel 'copter can be applied just the same. All we have to do is allow for the fact that the wings of the bird or insect sweep out only a fraction p_s of the complete circle.

Fig. 3



Paddlewheel helicopter

$p_s = 1$



Hummingbird

$p_s = 0.8$

(Here we shall ignore the fact that, at the end of each sweep, the wings have to rotate longitudinally so as to present the correct angle on the return. Any effect of this rotation, whether beneficial or otherwise, can be simply incorporated into the factor p_s.)

How does this affect equations (6) to (10)? Instead of writing $\omega = 2\pi f$ we must write $\omega = 2\pi f.\ p_s$. Basically what this means is that, wherever you see f in an equation, you must replace it by $f.\ p_s$. In addition, it is more convenient to measure the total wing area S_W rather than the wing chord c so we can put $2bc = S_W$. The table below summarizes all the important relations for a hovering insect or bird with these improvements included.

$$F_{\text{lift}} = 4\pi^2/3\ b^2\ S_W\, \rho\, f^2\, p_s^2\ \sin^2(\alpha)\cos(\alpha) = Mg \tag{17}$$

$$f = \sqrt{\frac{3Mg}{4\pi^2 b^2 S_W\ \rho\ p_s^2 \sin^2(\alpha)\cos(\alpha)}} \tag{18}$$

$$\text{Power}_{\text{loss}} = 2\pi^3\, b^3\ S_W\, \rho\, f^3\, p_s^3\ \sin^3(\alpha) \tag{19}$$

$$R_{LP} = \frac{2\cot(a)}{3\pi\ b\ f\ p_s} \tag{20}$$

Applying these formulae to some common animals (and including the paddle 'copter for comparison) yields the following results for the required wingbeat frequency (*Fig.4*).

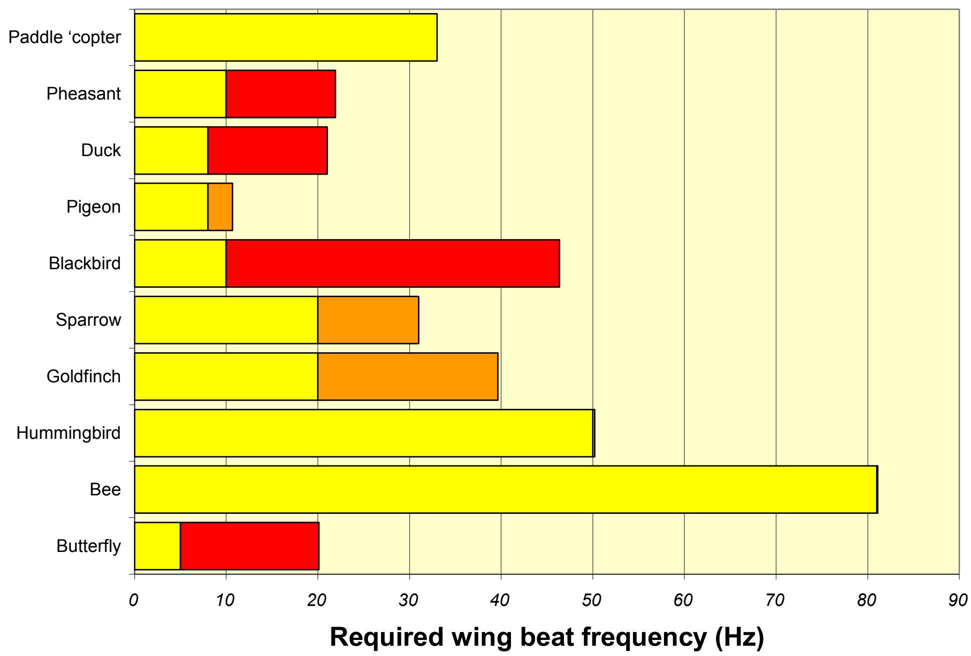


Fig. 4

Bars in yellow (the paddle 'copter, the hummingbird and the bee) indicate species which can obviously hover for extended periods and the predicted wingbeat frequencies are consistent with observations. For the others, the yellow part of the bar indicates the maximum observed wingbeat frequency, while the total length of the bar represents the theoretical required frequency. The excess is coloured orange or red. The pigeon, sparrow and goldfinch whose excess is coloured orange, all appear to be able to hover or achieve vertical flight for brief periods of time but the predicted wingbeat frequencies appear to be greater than those observed. It is true that none of these species can hover for long but we have all seen them doing it, if only briefly, and their ability obviously needs some extra explaining. The pheasant, the duck, the blackbird and the butterfly cannot hover and it is pleasing to see that all these species show a satisfactorily large excess frequency (coloured red).

What about the butterfly? Surely that can hover? I submit that it cannot. If you observe a butterfly settling on a flower, I think you will agree that either it is facing head to wind or it lands with a significant forward velocity. To hover in stationary air, a butterfly would have to flap its wings 20 times a second and it probably does not have the muscle power to do this.

(A word of caution is in order here. Morphological data for the bird species has been taken from published data[3,4] but I have had to estimate values for the pronation angle α and the swept angle p_s as there is no published information on these parameters. Indeed, these are the very parameters which the creature will alter to suit the circumstances it finds itself in. A rigorous test of the theory would therefore have to be applied to a particular individual behaving under controlled laboratory conditions and not, as here, to some putative average over a whole species. The data for the observed maximum wingbeat frequency are my own and were made by trying to match a drum roll to the frequency of the birds wing. Obviously there is considerable scope for error here!)

The total length of the bars in *Fig. 5* indicates the theoretical power to mass ratio which must be achieved by an animal if it wishes to hover.

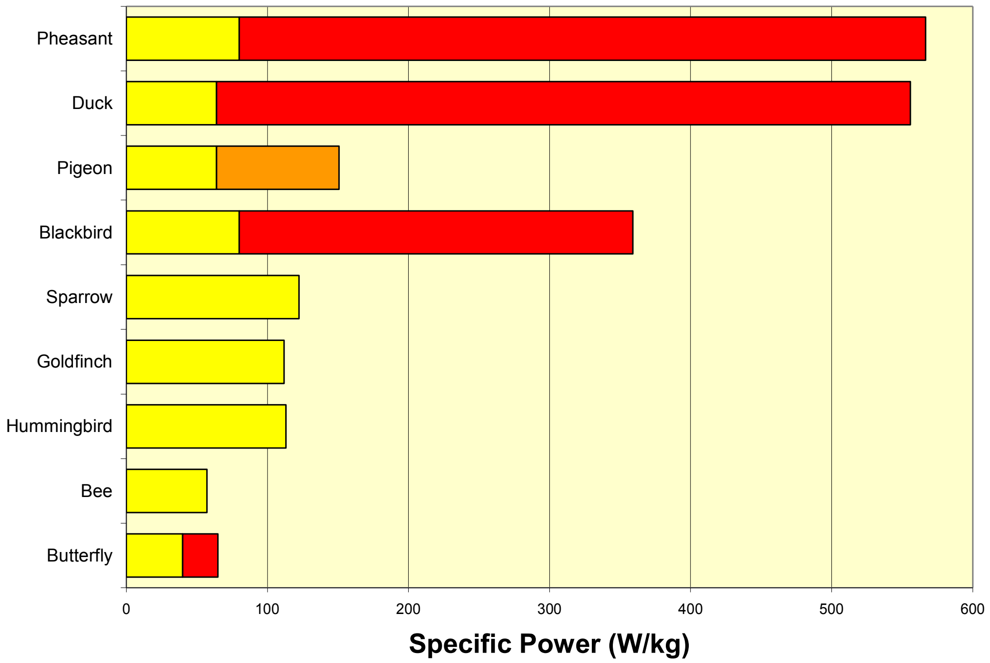


Fig. 5

Alexander[5] quotes a figure for the maximum energy output of striated muscle of 20 – 25 J kg^{-1} per wingbeat. I have used this figure, together with estimates of the ratio of muscle mass to body mass to calculate a figure for the maximum possible value for the Specific Power of each species. Where this figure is less than the calculated requirement, the difference is shown in red (if the bird cannot hover) and in orange if it is a borderline case. Once again, the theory suggests that the pigeon cannot hover (or fly vertically). The sparrow, goldfinch, hummingbird and bee are well within the permitted values, however.

It is of considerable interest to note that a human being of mass 80 kg can produce a maximum power output of about 1200 W from his leg muscles for a short period of time; that is a specific power output of just 15 W kg^{-1} – a figure which barely registers on the above chart! A suitably adapted human can only just fly, let alone hover!

Another interesting bird which can obviously hover but which I have not included is the Kestrel. The reason for this is that the Kestrel does not supinate its wings during hovering. It is basically flying into wind using ‘vectored thrust’ to provide additional lift at the low air speed.

Conclusions

So how bad is the simple theory? Can a bee fly? Yes it can. A bee is capable of up to 200 wingbeats per second. It will have no trouble managing 81. Nor does the hummingbird have any difficulty producing 50 wingbeats per second. It is true that the theory has difficulty explaining the abilities of the three borderline species pigeon, sparrow and goldfinch but to test the theory properly, much more accurate data on wing shape, pronation angle and swept ratio is needed. It is, in any case, at least within a factor of two and it correctly predicts the inability of birds like pheasants and ducks to hover. The remarkable issue to emerge is not the mechanism of lift production (which is basically just batting down as much air as you can as fast as you can!) but the prodigious amounts of power needed to sustain it. In other words, the problem is not with the Physics, it is with the Biology. How can a hummingbird produce nearly a watt of continuous power from a body that only weighs 6g? What special muscle adaptations are found in a pigeon which allow it, albeit very briefly, to indulge in vertical flight? How does the modest bee produce four times as much power, weight for weight, than a human being? These are the true miracles.

References

[1] **Ellington, C.P**. (1984b). The aerodynamics of hovering insect flight. I. The quasi-steady analysis. *Phil. Trans. R. Soc. Lond. B* **305**, 1-15

[2] **Sane, S.P.** (2003). The aerodynamics of insect flight. *J. of Exp. Biol.* **206** 4191-4208

[3] **Marden, J.H.** (1987). Maximum lift production during takeoff in flying animals. *J. of Exp. Biol.* **130** 235-258

[4] **Pennycuick, C.J.**, Speeds and wingbeat frequencies of migrating birds compared with calculated benchmarks, *The Journal of Experimental Biology* Vol. **204**, pp 3283-3294 (2001)

[5] **Alexander, R.M**. Principles of Animal Locomotion, *Princeton University Press*. p28 (Paperback edition)

An excel spreadsheet containing the data used to generate the charts illustrated above is available from the author. Please email jolinton@btinternet.com