Dynamic soaring

The Rayleigh cycle

In 1883 J. W. S. Rayleigh described how a bird such as an albatross could stay aloft for years at a time without once flapping its wings. At first sight, this appears to be impossible. Whenever a bird flies through the air energy is inevitably lost, not only by friction between its feathers and the air through which it moves but also because the very act of obtaining lift uses energy. If the bird uses no effort, all this energy must come from outside. But where? The obvious source is from the air itself; in short, the wind.

Many birds use vertical winds to keep them aloft. Gulls swooping round sea cliffs use the air deflected up over the cliffs to give them lift; a buzzard circling lazily in a rising thermal over a cornfield on a hot day is also using a vertical air current to gain height.

But what about an albatross flying huge distances over the vast southern ocean with neither cliffs nor thermals to assist? Well, the one thing the southern ocean has in abundance is – wind. But there is a problem here. Obviously a bird cannot extract kinetic energy from perfectly still air; but neither can it extract energy from uniformly moving air (because we can always choose a frame of reference in which the air is stationary.) What the bird exploits is the fact that the wind is never uniform. Sometimes it blows strongly, other times less so; there may be a squall over here but calm water over there; in between two waves the air is fairly still, but poke your head over the wave tops and it is blowing a gale.

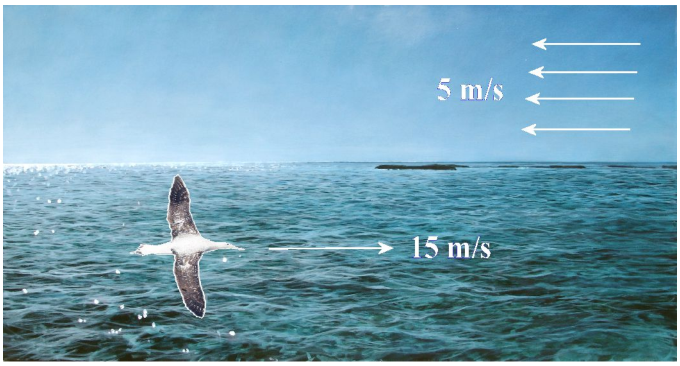
We refer to the situation where the wind speed differs from one place to another (either vertically or horizontally) as *wind shear* and where it differs from one moment to the next as a *wind gust*. Albatrosses can exploit both types of differential wind speed to gain energy and hence lift.

Extracting energy from wind shear

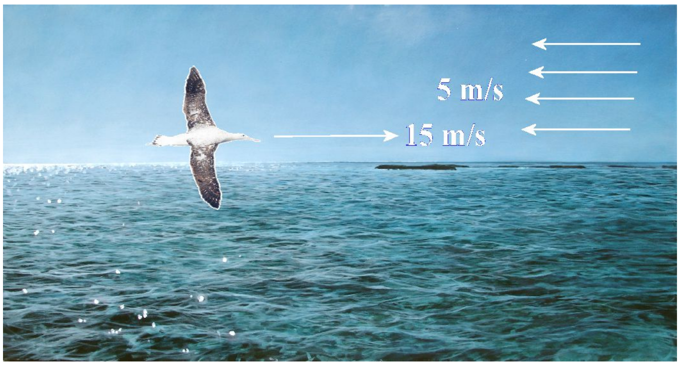
The most common form of wind shear is vertical. Air flowing over a choppy sea is retarded close to the surface of the sea and may be found blowing strongly at a height of a few metres. Let us consider an ideal situation where the velocity of the wind is zero below 2 m and 5 m/s above that height. Let us also suppose that our ideal albatross is capable of flying at any constant speed between 10 m/s and 30 m/s without loss of speed or height. (We shall, of course, consider more realistic assumptions in due course.)

One very important thing to realise is that such a bird can, in theory, bank its wings and turn 180° without any loss of airspeed. The reason for this is that the centripetal force needed to turn is always at right angles to the motion of the bird and therefore neither gives nor takes energy from the bird.

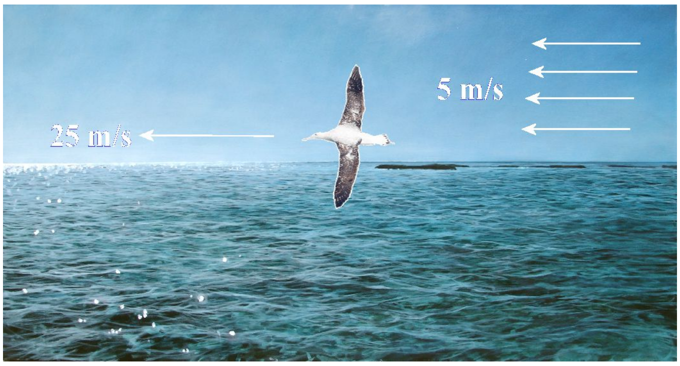
Now let us suppose that the bird is flying through the still air below the wind shear level at a speed of 15 m/s in a direction opposite to the wind above it like this:



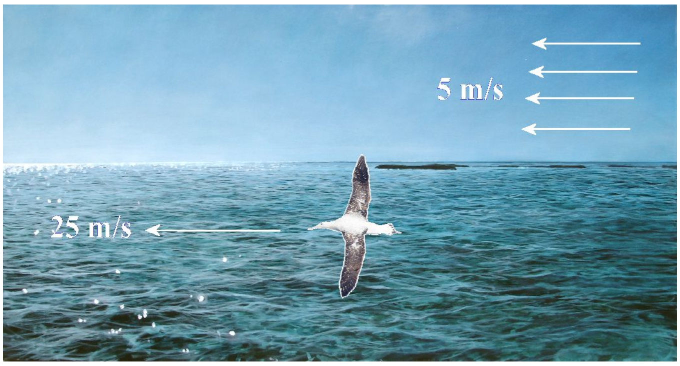
The bird now alters it wings in order to rise into the moving air. We assume that it can do this without loss of groundspeed so it now finds itself moving through the air with an airspeed of 20 m/s.



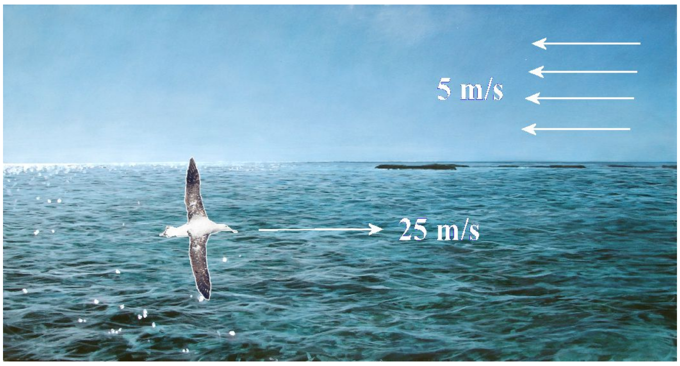
Next the bird executes a 180° turn downwind. Viewed from the point of view of the bird, he is just doing a normal turn at a constant airspeed of 20 m/s but viewed from the ground, the situation looks rather different. As the bird banks its wings, the moving air accelerates the bird downwind so that its groundspeed increases from 15 m/s to 25 m/s. (In the process, some of the moving air is slowed down and it is during this vital phase that energy is extracted from the wind.)



Now all the bird has to do to complete the manoeuvre is to drop down into the still air:



and make another 180° turn to end up travelling as before but significantly faster.



Instead of using a layer of faster air to accelerate, the bird can also use a wind gust. Suppose, as before, that the bird is travelling at 15 m/s in still air towards a 5 m/s gust heading towards him. When he reaches the gust, his airspeed increases to 20 m/s. Quickly he banks over to pick up speed from the gust until he is travelling downwind at 25 m/s (relative to the ground|). Now he must wait until the gust dies down before turning back onto his original heading. Clearly this method is not as useful as the wind shear method because the bird has to wait for the right conditions but no doubt an intelligent albatross, attuned as he is to every minute change in windspeed, will use every strategy available as an when it occurs.

The glide angle

We now turn to a more detailed analysis which takes into account what is known as the induced drag. This is the drag that inevitably occurs when lift is generated by a moving wing. The most important relations are the following:

$$F_L = \frac{1}{2} C_L S_W \rho v^2 \tag{1}$$

$$F_{ID} = \frac{1}{2} C_{ID} S_W \rho v^2 \tag{2}$$

where:

$$C_L = \frac{2\pi\alpha}{(1 + 2/A)} \tag{3}$$

and

$$C_{ID} \geq \frac{C_L^2 (1 + 2/A)}{4\pi} \tag{4}$$

F_L and F_{ID} are the forces of lift and induced drag respectively
C_L and C_{ID} are the coefficients of lift and induced drag
S_W is the effective wing area, and ρ is the density of air
v is the airspeed, A is the aspect ratio of the wing and α is the angle of attack

Equations (1) and (2) are basically just definitions. Equation (3) is a well established empirical equation as well as having sound theoretical justification. Equation (4) relies on considerations of the conservation of energy and may be regarded as a minimum expression for the coefficient of induced drag.

Eliminating C_L and C_{ID} we get:

$$F_L = \frac{\pi\alpha S_W \rho v^2}{(1 + 2/A)} \tag{5}$$

and

$$F_{ID} \geq \frac{\pi\alpha^2 S_W \rho v^2}{2(1 + 2/A)} \tag{6}$$

We can simplify these expressions by noting that, for any given bird the expression

$$\frac{\pi S_W \rho}{(1 + 2/A)} \tag{7}$$

is constant. Replacing this with the letter k we get:

$$F_L = k\alpha v^2 \tag{8}$$

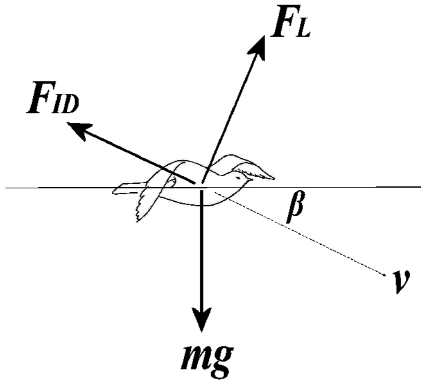
$$F_{ID} \geq \frac{1}{2} k\alpha^2 v^2 \tag{9}$$

It is also interesting to note that

$$\frac{F_{ID}}{F_L} \geq \frac{1}{2}\alpha \qquad (10)$$

(an important theorem in the theory of flapping flight).

Armed with this theory, we can now calculate the glide angle of any given bird in terms of its airspeed. Here is a diagram of the forces acting on a bird gliding down at an angle *β* at a constant speed *v*:



Since the forces are in equilibrium,

$$F_L = mg\cos\beta \qquad (11)$$

and

$$F_{ID} = mg\sin\beta \qquad (12)$$

From which we deduce that (assuming the usual approximations for small angles)

$$\beta \geq \frac{mg}{2k\,v^2} \qquad (13)$$

and

$$F_{ID} \geq \frac{m^2 g^2}{2k\,v^2} \qquad (14)$$

(It may be a bit surprising to see a formula which appears to say that the drag force *decreases* with increasing speed but the reason is simple. At the higher speed, the bird uses a much smaller angle of attack *reducing* the drag force.)

It has been noted that at a speed of 16 m/s, a wandering albatross has a maximum glide ratio of 21.2. This corresponds to a glide angle of 0.047 rad. It is of interest to see how closely our theoretical result approaches this reality.

An adult albatross has a mass of about 10 kg, a wingspan of 3.0 m and an aspect ratio of about 12 This gives it a wing area of 0.75 m^2. The density of air at sea level is 1.2 kg m^{-3} so using equation (7) we have:

$$k = \frac{\pi S_W \rho}{(1 + 2/A)} = 3.4\,\text{kg/m} \qquad (15)$$

So from equation (13)

$$\beta \geq \frac{mg}{2k\,v^2} = 0.056 \text{ rad} \qquad (16)$$

(This represents a glide ratio of 18. It is difficult to see how an albatross can achieve a glide ratio of 21.2, especially when you consider that our theoretical result above does not even take into account parasitic drag. A glide ratio of about 16 would seem to be more likely.)

Energy considerations

We can now calculate the time *T* within which the albatross must complete each Rayleigh cycle.

Suppose that the wind speed at sea level is zero and the windspeed in the upper layer is 5 m/s. At the start of the cycle, the bird (whose mass is 10 kg) is flying (upwind) with a speed 15 m/s and therefore has KE = 1125 J. At the end of the first turn, the bird is flying with an airspeed of 20 m/s and a speed of 25 m/s relative to the ground. The gain in kinetic energy is 3125 – 1125 = 2000 J. This amount of energy would lift the bird a height of 20 m.

If we assume a fairly constant glide ratio of 16, the maximum flying distance (through the air) will therefore be 320m which, at an approximate average airspeed of, say 16 m/s, would take 20 s. This compares well with observations on real birds