Conic Sections in Polar Coordinates

Conic sections (ie the circle, ellipse, parabola and hyperbola) have the following property – the distance between any point on the curve and a straight line (d) is a fixed multiple (a) of the distance between the point and the focus of the curve (r).



Suppose that the straight line is the line x = -(a + 1) and the focus of the curve is at the origin. (This will mean that all the curves pass through the point (-1,0) Then:

$$d = ar = a + 1 + r\cos\theta$$

from which we deduce that

$$r = \frac{a+1}{a-\cos\theta}$$

This generates all the conic sections as follows:

a = 0	straight line
0 < <i>a</i> < 1	hyperbola
a = 1	parabola
$1 < a < \infty$	ellipse
$a = \infty$	circle

(Negative values of *a* generate similar curves.)