## Conic Sections in Polar Coordinates

Conic sections (ie the circle, ellipse, parabola and hyperbola) have the following property - the distance between any point on the curve and a straight line $(d)$ is a fixed multiple $(a)$ of the distance between the point and the focus of the curve $(r)$.


Suppose that the straight line is the line $x=-(a+1)$ and the focus of the curve is at the origin. (This will mean that all the curves pass through the point $(-1,0)$ Then:

$$
d=a r=a+1+r \cos \theta
$$

from which we deduce that

$$
r=\frac{a+1}{a-\cos \theta}
$$

This generates all the conic sections as follows:

$$
\begin{aligned}
a & =0 & \text { straight line } \\
0<a & <1 & \text { hyperbola } \\
a & =1 & \text { parabola } \\
1<a & <\infty & \text { ellipse } \\
a & =\infty & \text { circle }
\end{aligned}
$$

(Negative values of $a$ generate similar curves.)

