The Physics of Flight (2) - Flapping Wings

J. Oliver Linton
E-mail: jolinton@btinternet.com

Abstract

In any conventional aircraft the wings provide the lift and the engines provide the thrust. In a bird, however, the wings have to provide the thrust as well as the lift. In this article we shall use the ideas presented in the first article of this series to develop a simple theory to explain how this comes about, and use it to predict some important characteristics of the flight of birds including their wingbeat patterns and the minimum power requirements.

Rates of wing flapping

It is apparent to anyone with eyes that large birds flap their wings more slowly than small birds. An obvious question to ask is what, if any, relations exists between wingspan B, wingbeat frequency f, wingtip amplitude (measured peak-to-peak) A_0 and airspeed v?

A recent paper by G.K.Taylor[1] et. al. suggests that, for a wide variety of animals (including fish as well as birds and insects), efficient cruising locomotion requires that a certain dimensionless number

$$\sigma = A_0 f / v \tag{1}$$

be in the range 0.2 – 0.4 and for birds in cruising flight it is almost always nearly equal to 0.2. (This number is called the **Strouhal** number after the Czech physicist Vincenc Strouhal (1850-1922)).

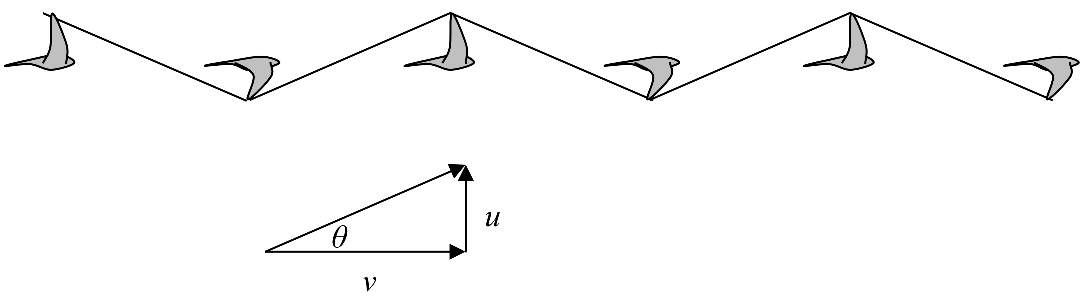
To keep things really simple, let us assume that the wing moves up and down at a constant speed u where

$$u = 2A_0 f \tag{2}$$

If the wing is moving forwards with a velocity v, the wingtip will be moving either up or down with a maximum 'zigzag' angle θ given by

$$\tan(\theta) = \frac{2A_0 f}{v} \tag{3}$$

Fig. 1



We have seen that the quantity $A_0 f / v$ is the Strouhal number σ so we can write

$$\tan(\theta) = 2\sigma \tag{4}$$

If σ = 0.2, θ, the Strouhal angle, is 22°.

What effect does this have on the flow of air over the wing and on the bird's ability to fly?

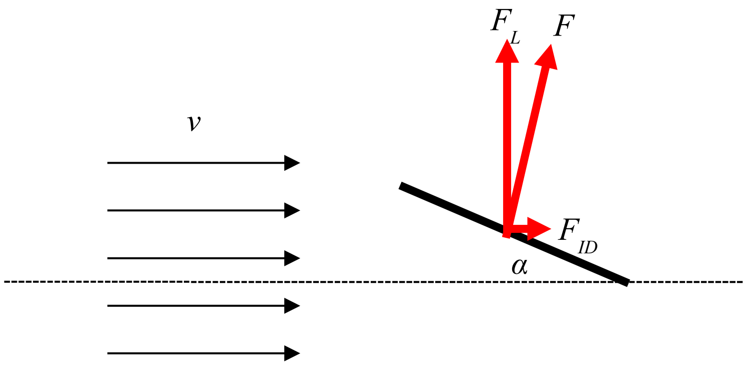
If a bird were to flap its wings in the vertical plane, up and down, keeping the wing chord horizontal all the time, the angle of attack (ie the angle at which the wing hits the oncoming air) would be 22° on the downstroke and -22° on the upstroke. Now conventional aerodynamic theory tells us that lift is only obtained with angles of attack less than about 15°. At higher angles of attack, the air flow over the top of wing detaches and the wing stalls. It follows that our hypothetical bird is going to stall the wing on the downstroke and, if anything, generate negative lift on the upstroke. This bird is going to fly like a brick! Obviously something needs explaining here.

Basic aerodynamics

When a wing moves forward through still air, the air exerts a force on the wing. At a certain position for any wing, when the wing is approximately parallel to the air stream, no lift is generated; the force is entirely drag. This is known as 'parasitic drag' and other parts of the bird's body will also make their own contribution to the total parasitic drag. For the moment we shall ignore this force.

The line down the centre of the wing in this position is known as the 'line of zero lift'. (Note that I am using this line as my datum line, not the more converntional 'wing chord'. This avoids having to introduce a coefficient of lift at zero angle of attack.) Now if the wing is inclined to the air stream at an angle of attack α (measured with respect to the line of zero lift) a large force appears on the wing F.

Fig. 2



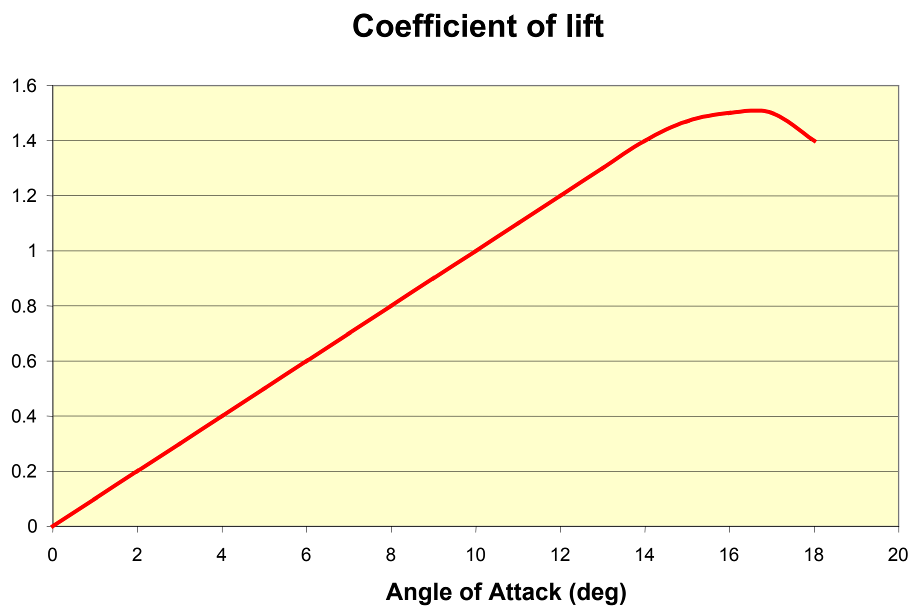
This force is resolved into two components, the lift F_L (which is at right angles to the flow of air) and the drag (known as 'induced drag') F_{ID}. Notice that, for a well designed wing, the force F is not at right angles to the wing but is inclined *forwards* with respect to the wing chord. This is crucial to understanding how a flapping wing generates forward thrust. In fact, as was shown in the first article, the angle between F and F_L –which is the reciprocal of the lift/drag ratio, is equal to $\alpha/2$.

It is usual to express the lift F_L in terms of a dimensionless constant called the aerodynamic coefficient of lift C_L by dividing the force F_L by the term ½ $S_W \rho\, v^2$. This takes into account the area S_W of the wing, the density ρ of the air (equal to 1.3 kg m^{-3} at sea level) and the velocity of the wing through the air v.

Hence $$F_L = \tfrac{1}{2}\, C_L S_W \rho\, v^2 \qquad (5)$$

Note that although I have called C_L a 'constant', it varies critically with angle of attack. Within the laminar region up to about, say, 15° C_L is roughly proportional to α. Beyond 15° the wing stalls and the lift drops rapidly. A graph of C_L against α looks something like this, though the precise details will depend crucially of the shape and construction of the wing.

Fig. 3



Using the arguments developed in my previous article on fixed and rotating wings, we deduced that

$$C_L = \frac{2\pi\,\alpha}{(1 + 2/A)} \tag{6}$$

where A is the aspect ratio of the wing. Most birds have aspect ratios in the region of 5 or 6 so we shall simplify our equations a bit by writing

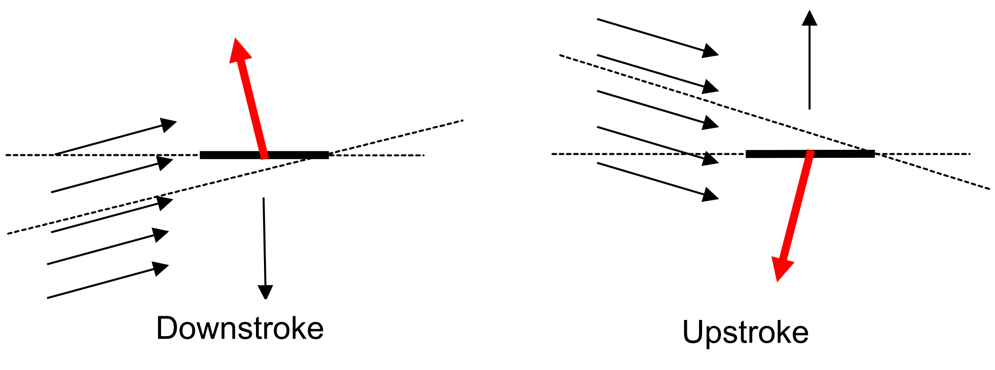
$$C_L \approx k_L\,\alpha \tag{7}$$

Where k_L is approximately equal to 5 and α in radians.

Generating thrust

When a bird in cruising flight flaps its wings, the flow of air over the wing is no longer horizontal. On the downstroke, the relative velocity between the air and the wing is inclined upwards from below, while on the upstroke, the velocity of the air is angled from above. Now, assuming for the moment that the wing is symmetrical and has zero angle of attack, it can be seen from the diagram below that on both the downstroke and on the upstroke, the force of lift on the wing (which is, by definition, at right angles to the flow of air) is inclined forwards and has a significant forward component. This is the origin of the thrust produced by a flapping wing.

Fig. 4

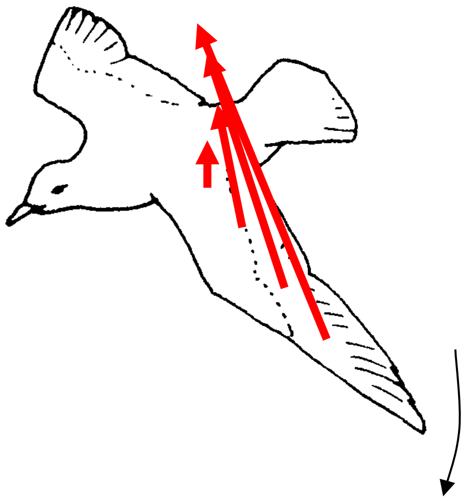


(It is equally clear that, on average, this wing will generate no lift – but that is because the angle of attack is zero. I have, for simplicity omitted the induced drag here. We shall consider the drag forces later. All that we require is that there shall be a net *forward* component of the lift.)

Now what is the answer to the objection that, if the Strouhal angle is 22° or more, the wing will stall on the downstroke? We must remember that a birds wing rotates about the shoulder and that the Strouhal angle is measured at the wing tip. The angle of attack will therefore vary along the wing from zero at the shoulder to a maximum of 22° at the tip. Most of the wing is still within the acceptable range of attack angles. The problem does, however, indicate that a vigorously flapping wing is probably going to generate less lift and less thrust than the theory would suggest.

We can now visualize the lift forces acting on a birds wing during the downstroke.

Fig. 5



As we move from the shoulder to the wing, the lift increases in size and inclines more and more forward. Since the angle of attack at the wingtip is equal to the Strouhal angle, the angle of attack α at a distance r from the wing root is

$$\alpha = \frac{2A_0 f}{v} \times \frac{r}{b} \qquad (8)$$

where b is the semi-span of the wing.In order to calculate the total forward thrust, we are going to have to integrate along the length of the wing from 0 to b for each wing. We must also remember that the resultant of the lift and drag forces inclines forward at an angle of α/2 not α. Hence

$$\text{Thrust} = 2\int_0^b \tfrac{1}{2} C_L \; c \; \rho \; v^2 dr \times \alpha/2 \qquad (9)$$

where c is the mean wing chord. Substituting $C_L = k_L \alpha$ gives

$$\text{Thrust} = \int_0^b \tfrac{1}{2} k_L \alpha^2 c\rho \; v^2 dr = \frac{2A_0{}^2 f^2 k_L c\rho}{b^2} \int_0^b r^2 dr \qquad (10)$$

Putting $S_W = 2bc$, this works out to be

$$\text{Thrust} = \tfrac{1}{3} k_L S_W \; \rho \; A_0{}^2 f^2 \qquad (10)$$

or, more conveniently in terms of the Strouhal number σ

$$\text{Thrust} = \tfrac{1}{3} k_L S_W \; \rho \, \sigma^2 v^2 \qquad (11)$$

Let us see what the formula (11) predicts.

A wood pigeon has a body mass of 0.5 kg, a wing span of 0.75 m and a total wing area of 0.08 m^2. It cruises at 15 ms^{-1} and flaps its wings 6 times every second. (This data is taken from Pennycuick[2]). We shall take k_L to be equal to 5. and assume that the Strouhal number is 0.2. Using these figures, the thrust generated by the pigeon is 1.6 N which is one third of the weight of the animal – a surprisingly large figure. As we have noted above, this result must be taken with a pinch of salt. The integral suggested above takes no account of the planform shape of the wing, nor does it allow for the possible loss of lift, and hence thrust, at the wingtips. In fact, since both the magnitude and the inclination of the lift force is greatest at the wingtips, any loss here is going to have a significant affect on the thrust generated. More will be said about this later.

Generating lift

Of course, in order to fly straight and level, the wing must also produce a net lift. It does this by using a positive angle of attack β. On the downstroke, this angle is added to the α term, which governs the size of the lift force, but it does not change its angle with respect to the vertical. Also, since the vertical component of the lift force depends on the cosine of α, we shall ignore this factor as α is small. The expression we need to integrate is therefore:

$$\text{Lift} = 2\int_0^b \tfrac{1}{2} k_L (\pm\alpha + \beta) c\rho\, v^2 dr = k_L c\rho\, v^2 \int_0^b (\pm\alpha + \beta) dr \qquad (12)$$

Because the angles are added together, the two parts can be integrated separately. Moreover, since the component due to the flapping reverses sign on the upstroke (indicated by the ± sign), this component cancels out leaving the mean lift on the wing unchanged at

$$\text{Lift} = \tfrac{1}{2} k_L \beta\, S_W\, \rho\, v^2 \qquad (13)$$

Likewise, when performing a similar calculation for the thrust, it is the lift component which reverses in sign leaving the thrust largely unchanged.

There is, however, the problem of whether or not the wing generates negative lift on the upstroke. Mathematically we have tacitly assumed that the relation between coefficient of lift and angle of attack (6) works both for angles in excess of 15° and for negative angles as well. This is simply not the case. It is true that immense amounts of lift (and thrust) are generated on the downstroke but it is inevitable that, on the upstroke, the highly cambered wing of a typical bird is going to stall and the forces generated, both thrust and the – probably negative – lift are going to be much reduced. When a large bird such as a heron with relatively slow wingbeats flies close past, you can actually hear the powerful thrust of the downbeat but the upstroke is relatively quiet. Many large birds partially fold their wings during the upstroke reducing both the area of the wing and, even more importantly, the vertical speed of the wing tip. I have also observed crows using an asymmetrical wingbeat – a long powerful downstroke followed by a quick upstroke. This is further evidence that in many situations, the forces during the upstroke are considerably smaller than those involved in the downstroke. I have also noticed that many soaring birds like gulls and birds of prey do not simply flap their wings up and down but use an elliptical motion which carries the wing backwards during the downstroke and forwards during the upstroke. This changes little on the downstroke but significantly decreases the (negative) angle of attack on the upstroke, possibly even reducing it to zero.

But perhaps the most important fact to be born in mind is that, unlike the wing we are considering in our model, a bird's wing is not rigid. On the downstroke, the pressure of the air underneath the wing tips (where the angle of attack is greatest) bends the trailing edge of the primary feathers upwards, preventing a stall, while on the upstroke, any excess pressure on the top of the wing flexes the trailing edge downwards, ensuring that the angle of attack never goes much below zero.

The upshot of this discussion is that, in many cases, we can probably assume that when flapping vigorously, the formulae we have derived work reasonably well for the downstroke but that a bird typically gains neither lift nor thrust during the upstroke. The mean thrust and lift will therefore be approximately halved, ie:

$$\text{Mean thrust} = \tfrac{1}{6} k_L S_W\, \rho\, \sigma^2 v^2 \qquad (14)$$

$$\text{Mean lift} = \tfrac{1}{4} k_L \beta\, S_W\, \rho\, v^2 \qquad (15)$$

Cruising flight

In cruising flight at a constant level speed, the mean thrust must be equal to the total drag on the bird. This can be estimated in a very simple way. When a pigeon stops flapping its wings and glides, it loses about 1 m in height for every 8 m traveled horizontally; in other words, it has a *glide ratio* (R_G) of about 0.125. It is easy to see that this must also be the ratio between the total drag on the bird and its weight. So, if the bird weighs 5 N, the total drag while cruising will be around 0.625 N. (Note that this method of calculating the total drag includes both parasitic and induced drag.) Equation (14) allows us to calculate the expected Strouhal number. For a pigeon it works out to be 0.18. While not quite as close to the observed value of 0.2 as I would wish, the agreement is sufficiently close to suggest that the analysis presented above is not a million miles from the mark. The error is at least on the right side of the line as it is much easier to think of reasons why the bird will in practice have to flap its wings *more* vigorously than the theory predicts rather than less.

For a bird in level cruising flight, equations (14) and (15) can be rearranged in terms of the birds mass as follows:

$$\text{Strouhal number: } \sigma = \sqrt{\frac{6Mg\, R_G}{k_L S_W \rho\, v^2}} \qquad (16)$$

$$\text{Angle of attack: } \beta = \frac{4Mg}{k_L S_W \rho\, v^2} \qquad (17)$$

Another interesting parameter which we can now derive is the flap angle φ. This is the total angle through which the bird's wing moves from top to bottom. It is easy to see that

$$\phi = \frac{A_0}{b} = \frac{\sigma v}{f\, b} = \sqrt{\frac{6\, Mg\, R_G}{k_L S_W f\, b \rho}} \qquad (18)$$

A crucial test of the theory presented here will be to see if this expression generates plausible flap angles – eg less than 180 degrees!

The chart below shows the predicted Strouhal numbers and Flap angles for a number of different species. I have had to estimate the glide ratios for the different species as this important characteristic does not seem to have been measured by any of the researchers whose works I have consulted. The birds are in decreasing order of mass.

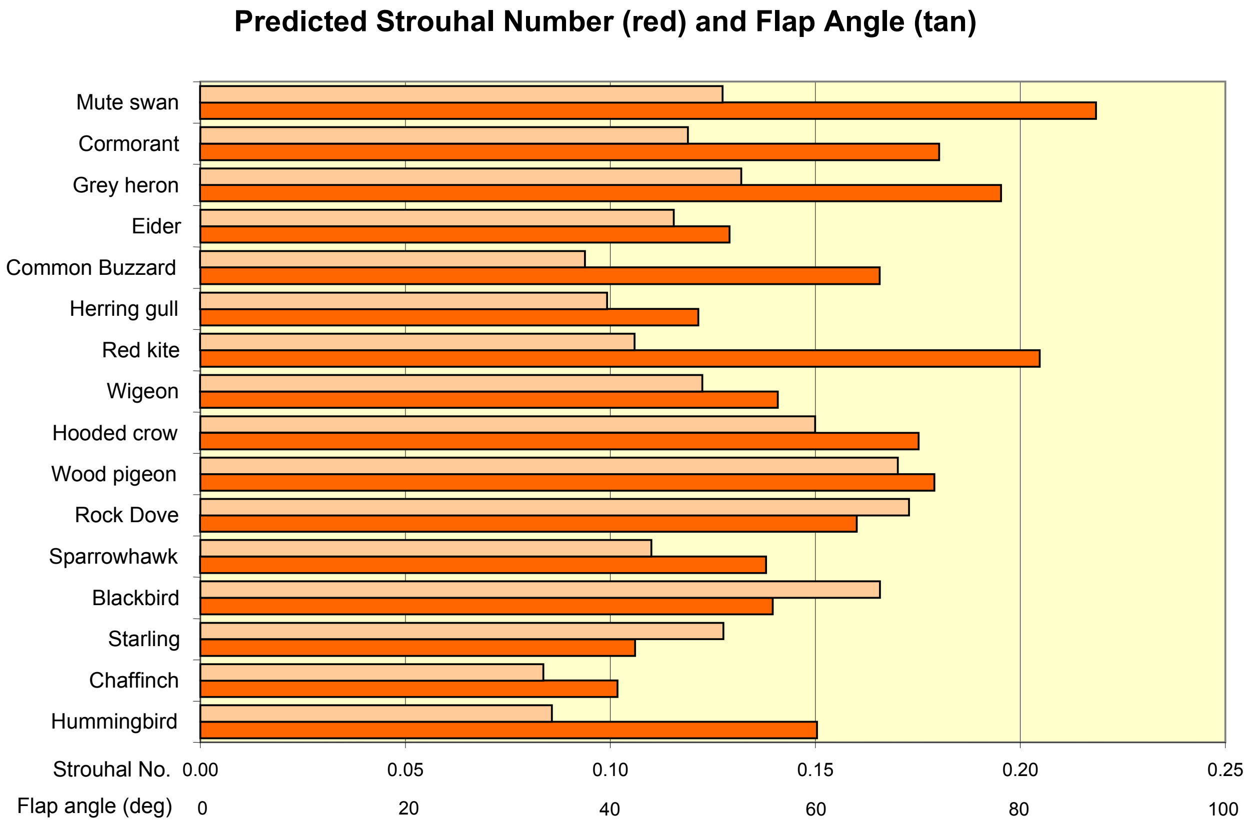


Fig. 6

As previously noted, the predicted Strouhal numbers are marginally lower than we would expect but it is interesting to note that the number is indeed largely independent of the mass of the bird. (a swan is three orders of magnitude heavier than a hummingbird).

Gratifyingly, the predicted flap angles are emminently plausible (albeit slightly lower than one might expect). Indeed they are remarkably consistent with only the crow, the pigeons and the blackbird having to exert themselves more than the average.

Power considerations

Finally, how much power does it take to fly?

The work done in pushing the bird through the air is simply the Thrust × Speed. ie:

$$\text{Power} = \tfrac{1}{6} k_L S_W \ \rho \, \sigma^2 v^3 \tag{19}$$

or, more easily still in terms of the glide ratio:

$$\text{Power} = M \, g \, R_G \, v \tag{20}$$

For a pigeon this works out to be 9.4 W. (Note that this latter expression is only valid for cruising speeds and typically represents the *minimum* power needed to stay airborn.)

Figure 6 shows the power requirements of various species expressed in terms of body mass.

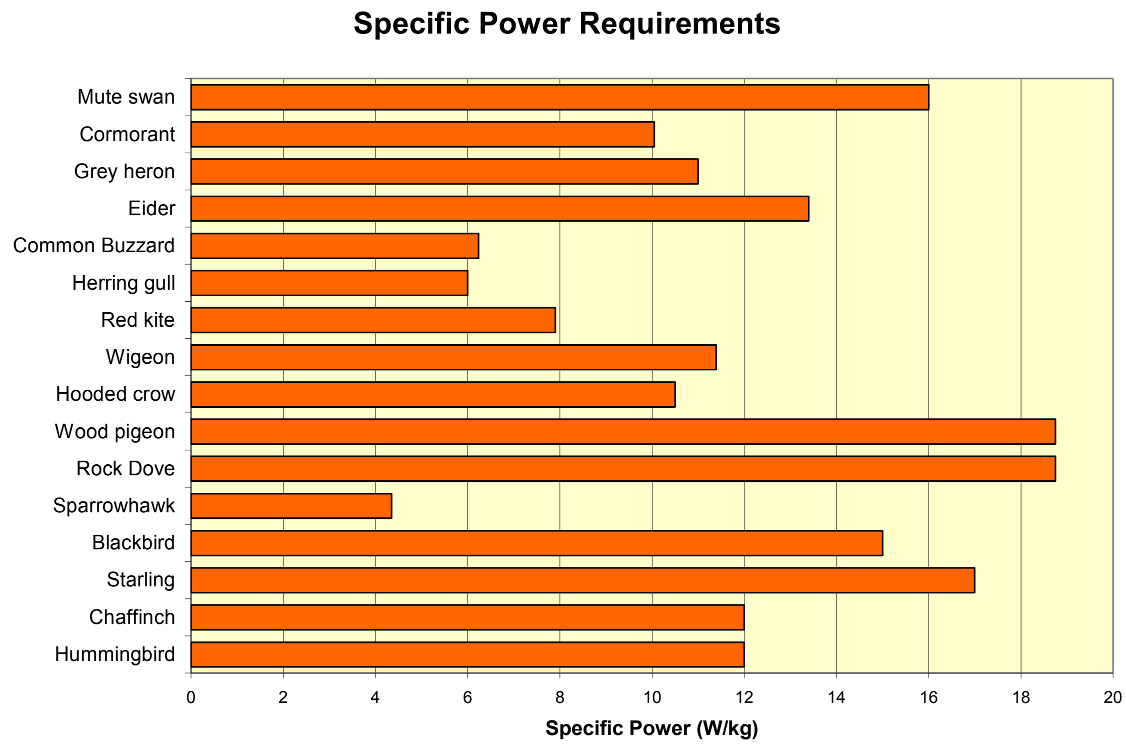


Fig. 7

It is interesting to see that, because of their streamlined shapes and low glide ratios, birds like gulls and hawks have relatively small power requirements. For comparison, it is worth noting that a human being of typical mass 80 kg can produce a maximum continuous power output of about 200W – ie a specific power output of 2.5 Wkg^{-1}. In 1979 an extremely fit amateur cyclist, Bryan Allen managed to fly across the channel in an extraordinary flying machine called the Gossamer Albatross. A pigeon can produce 7 times as much specific power.

Conclusions

The aerodynamics of flapping flight has, naturally, received a lot of attention over the years. Pennycuick[3] published the first comprehensive analysis in the 1960's using a quasi-steady analysis similar to the one developed above. Since then, Rayner[4] and others have developed more sophisticated, unsteady analyses based on the dynamics of vortex rings and it is now possible to carry out very sophisticated computer calculations on flapping wings. It is not my intention to compete with these illustrious names but merely to show how the application of simple A level physics can provide a basic understanding of how flapping generates thrust and to make semi-quantitative predictions which are at least consistent with everyday observations of birds in flight.

[1] **Taylor, G.K., Nudds, R.L., Thomas, A.L.R.**, "Flying and swimming animals cruise at a Strouhal number tuned for high power efficiency", *Nature* Vol. **425**, pp 707–711 (October 2003)

[2] **Pennycuick, C.J.**, "Speeds and wingbeat frequencies of migrating birds compared with calculated benchmarks", *The Journal of Experimental Biology* Vol. **204**, pp 3283-3294 (2001)

[3] **Pennycuick, C.J.**, "Power requirements for horizontal flight in the pigeon Columba livia", *Journal of Experimental Biology*, Vol. **49**, No. 3, pp 527-555, (1968)

[4] **Rayner, J.M.V.**, "A new approach to Animal Flight Mechanics", *Journal of Experimental Biology*, Vol. **80**, No. 1, pp 17-54, (1979)

An excel spreadsheet containing the data used to generate the charts illustrated above is available from the author. Please email jolinton@btinternet.com